Closing Thur: HW 14.2 (part 1)
Closing *next* Tue: HW 14.2 (part 2)
Closing *next* Thur: HW 14.3/4 (last HW)
Final: Sat, March 10, 5:00-7:50pm,
PAA Building

Entry Task: Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for

$$z = 5\ln(x) - \underbrace{x^4}_{F}\underbrace{y^2 e^{3x}}_{S} - \frac{4}{y^2}$$

$$\boxed{\frac{\partial z}{\partial x} = 5\cdot\frac{1}{x} - 3x^4y^2e^{3x} - 4x^3y^2e^{3x} - 0}$$

$F = x^4$ $\quad S = y^2e^{3x}$

$F' = 4x^3$ $\quad S' = y^2e^{3x}\cdot 3$

$$z = 5\ln(x) + x^4y^2e^{3x} - 4y^{-2}$$

$$\boxed{\begin{aligned}\frac{\partial z}{\partial y} &= 0 + x^4\,2y\,e^{3x} + 8y^{-3}\\ &= 2x^4ye^{3x} + \frac{8}{y^3}\end{aligned}}$$

Interpreting as a rate

Your company produces and sells **two** products (hats and sunglasses)

$x =$ number of hats

$y =$ number of glasses

You find that profit is given by

$$P(x, y) = -3x^2 + 30x - 5y^2 + 130y + 2xy - 100$$

1. Find the partial derivatives.
2. Find and interpret $P_x(5,8)$ and $P_y(5,8)$.

$P_x = -6x + 30 + 2y$

$P_y = -10y + 130 + 2x$

$P_x(5, 8) = -6(5) + 30 + 2(8) = 16 = \frac{\partial P}{\partial x}$ ← dollar in profit per ← hats

"The sale of the next hat after (5, 8) will increase profit by about \$16"

$P_y(5, 8) = -10(8) + 130 + 2(5)$
$= -80 + 130 + 10 = 60 = \frac{\partial P}{\partial y}$ ← dollars in profit per ← sunglasses

"The sale of the next sunglasses will increase profit by about \$60"

3. Estimate the values of

$$\frac{P(5.001,8) - P(5,8)}{0.001} \approx P_x(5,8) = 16$$

x changing

$$\frac{P(5, 8.01) - P(5,8)}{0.01} \approx P_y(5,8) = 60$$

y-changing

From HW: (Cobb-Douglas Model)

Q = units produced

K = capital expenditures (in thousand dollars)

L = hours of labor

$$Q = 75K^{1/3}L^{2/3}$$

Assume there are:

$2,744,000 in capital expenditures and 4913 in total labor hours.

Find and interpret $\frac{\partial Q}{\partial K}$ and $\frac{\partial Q}{\partial L}$.

$$\frac{\partial Q}{\partial K} = 75 \cdot \frac{1}{3} K^{-2/3} L^{2/3} = 25\, K^{-2/3} L^{2/3}$$

$$\frac{\partial Q}{\partial L} = 75\, K^{1/3} \frac{2}{3} L^{-1/3} = 50\, K^{1/3} L^{-1/3}$$

$K = 2744$ thousand dollars

$L = 4913$ hours

$$\frac{\partial Q}{\partial K} = 25\,(2744)^{-2/3}(4913)^{2/3}$$

≈ 36.8622

≈ 37 units/thousand dollars

INCREASE CAPITAL EXPENDITURES BY 1 thousand dollars leads to an increase in units produced of about 37.

$$\frac{\partial Q}{\partial L} = 50\,(2744)^{1/3}(4913)^{-1/3}$$

≈ 41.1765

≈ 41 units/labor hour

INCREASE LABOR HOURS BY 1

$\Rightarrow$ INCREASE IN UNITS BY 41.

Definition

A point (a,b) is a ***critical point*** for a function $z = f(x, y)$ if BOTH

$$f_x(a,b) = 0 \quad \textbf{and} \quad f_y(a,b) = 0.$$

ASIDE

MAX PROFIT

$= P(10,15)$

$= \cdots = \$1025$

Going back to our last example:

$$P(x, y) = -3x^2 + 30x - 5y^2 + 130y + 2xy - 100$$

Find the critical point.

$P_x = -6x + 30 + 2y \overset{?}{=} 0$ AND $P_y = -10y + 130 + 2x \overset{?}{=} 0$

$\Rightarrow 2y = 6x - 30$

$\Rightarrow y = 3x - 15$

$-10(3x-15) + 130 + 2x \overset{?}{=} 0$

$\Rightarrow -30x + 150 + 130 + 2x \overset{?}{=} 0$

$\Rightarrow 280 - 28x = 0$

$280 = 28x$

$\boxed{x = 10}$

$y = 3x - 15$

$y = 3(10) - 15 = 15$

CHECK!!!

ONLY CRITICAL POINT $(x,y) = (10,15)$

Graphical Interpretation

Pretend you are skiing on the surface

$z = f(x, y) = 15 - x^2 - y^2$

1. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$

2. Find and interpret
   $f_x(7,4)$ and $f_y(7,4)$

3. Find the critical point.

$$\frac{\partial z}{\partial x} = -2x \Rightarrow \frac{\partial z}{\partial x} = -14 \approx \frac{f(7.0001, 4) - f(7,4)}{0.0001} = \text{slope}$$

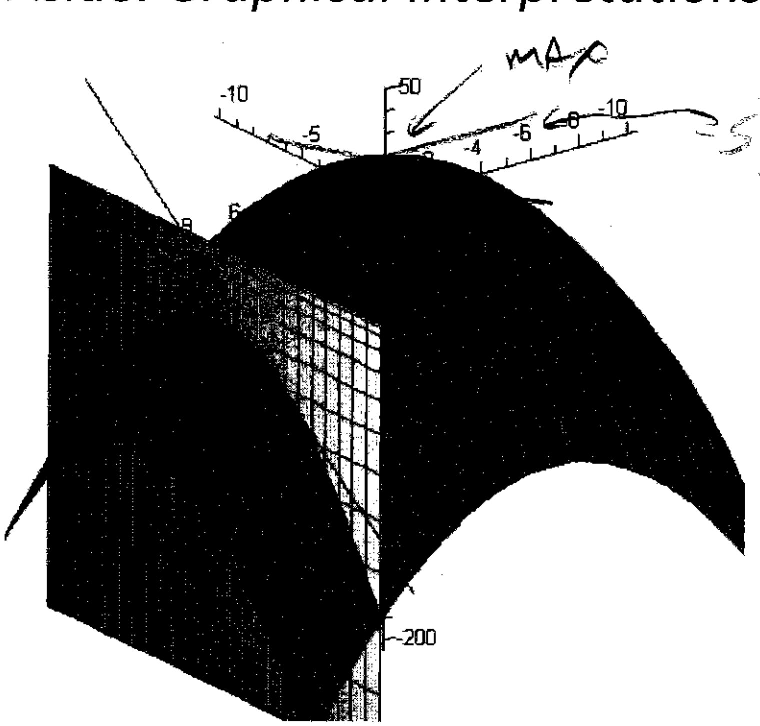
$$\frac{\partial z}{\partial y} = -2y \Rightarrow \frac{\partial z}{\partial y} = -8 \approx \frac{f(7, 4.00001) - f(7,4)}{0.00001} = \text{slope}$$

$-2x = 0$ $\underline{\text{AND}}$ $-2y = 0$

$\Rightarrow x = 0$ $\underline{\text{AND}}$ $y = 0$

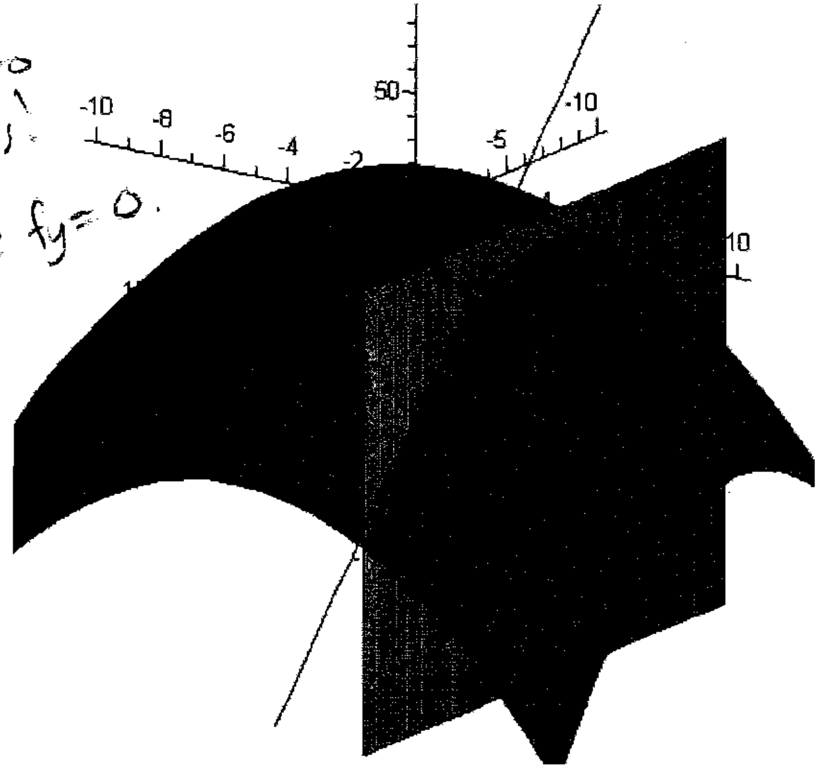
$$\boxed{\begin{array}{l}(0,0) \\ f(0,0) = 15 \leftarrow \text{MAX}\end{array}}$$

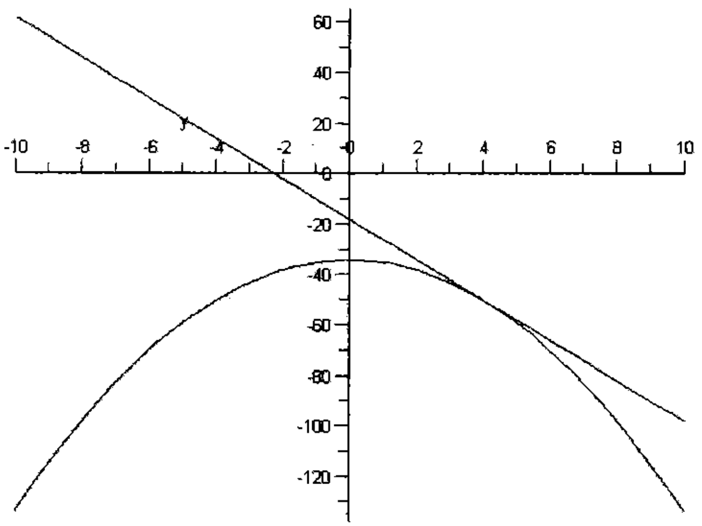
Aside: Graphical Interpretations

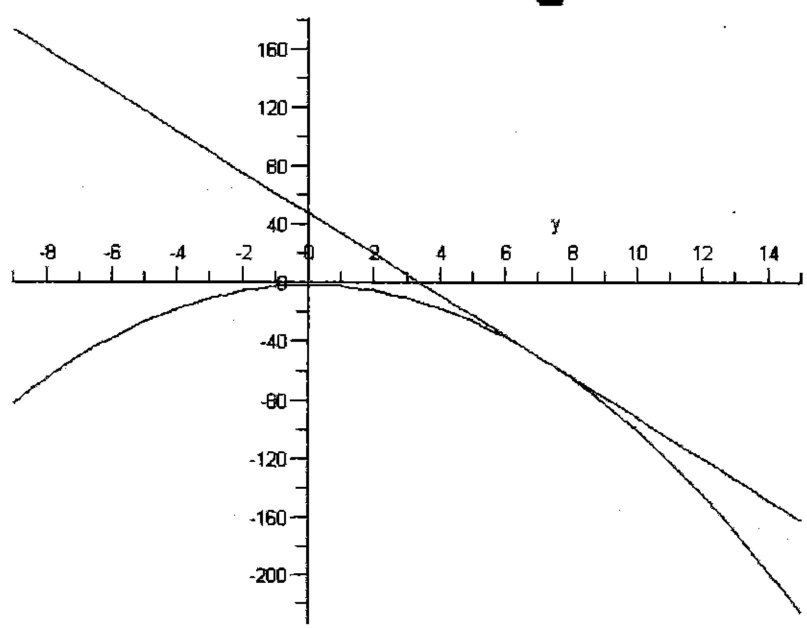


max

slope is zero both directions!
At top
$f_x = 0$ and $f_y = 0$.







Example:

Find all critical points of

$$f(x,y) = 2x^4 + y^2 - 4xy + 1$$

$f_x = 8x^3 - 4y \stackrel{?}{=} 0$ AND $f_y = 2y - 4x \stackrel{?}{=} 0$

$\Rightarrow \quad 8x^3 = 4y$

$\Rightarrow \quad y = 2x^3$

$2(2x^3) - 4x \stackrel{?}{=} 0$

$4x^3 - 4x \stackrel{?}{=} 0$

$4x(x^2 - 1) \stackrel{?}{=} 0$

$x = 0$ or $x^2 - 1 = 0$

$x = -1 \qquad x = 1$

3 possibilities

$x = 0 \Rightarrow y = 2(0)^3 = 0 \Rightarrow (x,y) = (0,0)$

$x = -1 \Rightarrow y = 2(-1)^3 = -2 \Rightarrow (x,y) = (-1,-2)$

$x = 1 \Rightarrow y = 2(1)^3 = 2 \Rightarrow (x,y) = (1,2)$

$\boxed{(x,y)=(0,0),\ (x,y)=(-1,-2),\ (x,y)=(1,2)}$

Check!!